MANAV RACHNA
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MANAV RACHNA UNIVERSITY
(FORMERLY MANAV RACHNA COLLEGE OF ENGINEERING
NAAC ACCREDITED 'A' GRADE INSTITUTION)
Declared as State Private University under section 2f of the UGC act, 1956

DEPARTMENT OF MATHEMATICS

"T3, MAY 2018"

Semester: IV
Subject:Struc.on Differentiable Manifold
Branch: M.Sc(Maths)
Course Type: Core
Time: 3 Hours
Max.Marks: 100

Date of Exam: 15/05/2018
Subject Code: MAH 631
Session: Even
Course Nature: Hard
Program: M.Sc
Signature: HOD/Associate HOD:

Part A:- Attempt any two Questions. Part B & Part C:Attempt any two questions from each part.

Part A

Q.1.(a) Determine the integral curve for the vector fields in R^2 given by $X = \frac{x^1 \partial}{\partial x^1} + x^2 \frac{\partial}{\partial x^2}$. (5)

b) If X and Y are differentiable vector field in R^2defined by $X = (x^1)^2 \frac{\partial}{\partial x^1}$, $Y = \frac{\partial}{\partial x^1} + \frac{(x^2)^2 \partial}{\partial x^2}$. (5)
Compute [x,y].

Q.2.(a) Show that for a given arbitrary connection B, connection D defined by $2D_x Y = B_x Y - \overline{B_x \bar{Y}}$ is a F-connection. (5)

b) Show that an affine connection D on an almost complex manifold is a F-connection if and only if
$D_X \bar{Y} = \overline{D_X Y}$. (5)

Q.3.(a) Let T be the torsion tensor of the F-connection D. Then show that $T(X,Y) - T(\bar{X},\bar{Y}) = \frac{1}{2} N(X,Y)$. (5)

b) If $X = xy\frac{\partial}{\partial x} + x^2\frac{\partial}{\partial z}$, $Y = y\frac{\partial}{\partial y}$ are vector field on R^3 and the map $f: R^3 \to R$ be defined by $f(x,y,z) = x^2y$. Find the value of $(Xf)_{(1,1,0)}$. (5)

Part B

Q.4.(a) Show that any affine connection ∇ on a smooth manifold M can be decomposed into a sum of a multiple of its torsion tensor and a torsion free connection. (10)

b) If $F'(X,Y) = g(\bar{X},Y)$, then show that ,
i) $F'(X,Y) = -F'(Y,X)$
ii) $F'(\bar{X},\bar{Y}) = F'(X,Y)$
iii) $F'(\bar{X},Y) = -F'(\bar{X},\bar{Y}) = -g(X,Y) \quad \forall X,Y \in TM$. (10)

Q.5.a) Let M be an almost Hermitian manifold with almost complex structure J and Hermitian metric g. Then show that the covariant derivative ∇ of the Riemannian connection defined by g , the fundamental 2-form and the torsion N of J satisfy

$$2g\big((\nabla_X J)Y, Z\big) - g\big(JX, N(Y,Z)\big) = 3d\phi(X,JY,JZ) - 3d\phi(X,Y,Z)$$

for any vector fields X, Y, Z on M. (10)

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b) Let M be an almost complex manifold with almost complex structure J and Hermitian metric g. Prove the following equivalence

i) $\nabla \bar{J} = 0.$

ii) $\nabla \phi = 0.$ (10)

Q.6.a) Let M be an almost complex manifold with almost complex structure J. Prove that M is a complex manifold if and only if M admits a linear connection ∇ s.t $\nabla J = 0$ and $T = 0$, where T denotes the torsion of ∇. (10)

b) Prove that an almost Hermitian manifold is a Kaehler manifold if and only if $\nabla_X \bar{Y} = \overline{\nabla_X Y}$. (10)

Part C

Q.7.a) Show that for a Kaehler manifold the following relation holds

i) $R(X,Y)\,\bar{Z} = \overline{R(X,Y)Z}$

ii) $\overline{R(X,Y)\,\bar{Z}} = -R(X,Y)Z.$ (10)

b) Show that the Ricci tensor S of a Kaehler manifold M satisfy

$$(\nabla_Z S)(X,Y) = (\nabla_X S)(Y,Z) + (\nabla_{jY} S)(JX, Z).$$ (10)

Q.8.a) Show that for a Kaehlerian manifold $S(JX,JY) = S(X,Y)$ and $S(X,Y) = \frac{1}{2}(trace\ of\ JR(X,JY))$ for any vector fields X and Y on M. (10)

b) Let M be a real 2n-dimensional Kaehler manifold. Show that if M is of constant curvature then M is flat provided n>1. (10)

Q.9.a) Show that for any quaternion Kaehlerian manifold M of dimension n =4m(m>1) , the Ricci tensor S of M is parallel. (10)

b) Show that any quaternion Kaehler manifold of $dim \geq 8$ is an Einstein manifold. (10)

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