MANAV RACHNA vidyanatarikasha

MANAV RACHNA UNIVERSITY

(FORMERLY MANAV RACHNA COLLEGE OF ENGINEERING NAAC ACCREDITED 'A' GRADE INSTITUTION)

Declared as State Private University under section 2f of the UGC act, 1956

DEPARTMENT OF MATHEMATICS

"T3-Examination, May-2018"

Semester: 4^{th}

Date of Exam: 23/05/2018

Subject: Probability & Mathematical Statistics

Subject Code: MAH 229-T

Branch: MATHEMATICS

Session: II

Course Type: Core

Course Nature: Hard

Time: 3:00 Hrs.

Program: B.Sc.

Max. Marks: 80

Amit Srivastava

Note:-All questions are compulsory from part –A (2× 10 = 20). Attempt any two questions from part B (15*Marks each*). Attempt any two questions from part C (15*Marks each*).

PART A

1(a) Define expectation of a function random .variable

(b) What is the least square value of prob$\{|X-5|<3\}$.

(c) Define marginal and conditional probability.

(d) Find the cumulative distribution function of X is F(x).Find the cumulative distribution function of

(i) $Y = X + a$ (ii) $Y = X - b$.

(e) From the following probability: (i) $P(X+Y)=8$ (ii) $P(X+Y)\geq 8$

(f) A probability curve $y=f(x)$ has range from 0 *to* ∞. if $f(x)=e^{-x}$. Find the *mean and varience.*

(g) Show that $|E(X)|\leq E|X|$.Provided the expectations exist.

(h) Write the theorem of moment generating function.

(i) Define Bivariate normal distribution.

(j) Let (X,Y) $\sim BVN(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho)$.Then X and Y are independent if and only if $\rho=0$.

PART B

2. (a) Write uniqueness theorem of moment generating function. [5]

(b) A random variable 'X' has probability function $f(x) = \frac{1}{2^x}$, $x = 1,2,3$

Find the moment generating function, mean and variance. [10]

3. If X is a random variable with mean μ and variance σ^2 then any positive number k, then so that

$P\{|X - \mu| \geq k\sigma\} \leq 1/k^2$ or $P\{|X - \mu| \geq k\sigma\} \geq 1 - 1/k^2$ [15]

4. Let X and Y are bivariate normal distribution with parameters $\mu_X = 5, \mu_Y = 10,$

$\sigma_X^2 = 1, \sigma_Y^2 = 25$ and $corr(X,Y) = \rho.$

(a) if $\rho > 0$ find ρ when $P(4 < Y < 16/X = 5) = 0.954$ (b) If $P(X + Y \leq 16)$ [15]

PART C

5. The joint probability distribution ot two random variables X and Y given by:

$$P(x,y) = \frac{2}{n(n+1)}; \quad x = 1,2,3,\ldots\ldots \text{ and } y = 1,2,3,\ldots\ldots x.$$

Examine whether X and Y are independent. [15]

6(a) Show that the expected value of X is equal to the expectation of the conditional expectation of X and Y.

Symbolically. $E(X) = E[E(X/Y)]$ [7]

(b) Let the joint probability density function of the random variable X and Y *be*

$$f(x,y) = \begin{cases} 2(x + y - 3xy^2); & 0 < x < 1 \\ 0, \text{otherwise}; & 0 < y < 1 \end{cases}$$. Find the marginal distribution of X and Y. [8]

7. Given $f(X/Y) = ne^{-x(y+1)}; \quad x \geq 0, y \geq 0$.Find the regression of Y on X. [15]