MANAV RACHNA vidyanatariksha | MANAV RACHNA UNIVERSITY
(FORMERLY MANAV RACHNA COLLEGE OF ENGINEERING
NAAC ACCREDITED 'A' GRADE INSTITUTION)
Declared as State Private University under section 2f of the UGC act, 1956

DEPARTMENT OF MATHEMATICS

"T3Examination, May2017-18"

Semester:6th
Subject:Linear Algebra
Branch: Mathematics
Course Type: Core
Time: 3 Hours
Max.Marks: 80

Date of Exam:18/05/2018
Subject Code:MAH339-T
Session: I
Course Nature: Hard
Program: B.Sc.
Signature: HOD/Associate HOD:

Note: All questions are compulsory from part A (2*10 = 20 Marks). Attempt any two questions from Part B (15 Marks each). Attempt any two Questions from Part -C (15 Marks each).

PART -A

Q.1 (a) Define quadratic form. Also give suitable example of a quadratic form.

(b) Write the matrix of Bilinear form given by: $2x_1y_1 + x_1y_3 - 2x_2y_1 + 7x_2y_2 - 2x_2y_3$. Also find rank of the above bilinear form.

(c) Define Inner Product space.

(d) Define orthonormal set with the help of a suitable example.

(e) State Parallelogram's law in an Inner Product Space.

(f) If $u = (1 + i, i, -1)$ and $v = (1 + i, 1 - i, 2i)$. Find Inner product of u and v.

(g) If $f: R^2 \times R^2 \to R$ defined by $f(X,Y) = x_1y_1 + x_2y_2$ for $X = (x_1, x_2) and\ Y = (y_1, y_2)$. Prove that f is a Bilinear form.

(h) Define Orthogonal complement of an Inner Product space.

(i) If V(F) is an inner product space, then prove that $V^{\perp} = \{0\}$.

(j) Define Adjoint operator of a linear transformation.

PART - B

Q.2 (a) Find all eigen values and corresponding eigen vectors of the linear operator given by:

$T: R^3 \to R^3$ defined by $T(x, y, z) = (x + y + z, 2y + z, 2y + 3z)$ (9)

(b) Prove that a quadratic form remains a quadratic form when subjected to a linear transformation. (6)

****

Q.3 (a) Reduce the following quadratic form into canonical form and also find equations of transformations. Also find rank, index and signature (10)

$x_1^2 + 3x_2^2 + 5x_3^2 + 4x_1x_2 + 4x_1x_3 + 10x_2x_3$

(b) Prove that quadratic form $ax^2 + 2hxy + by^2$ is positive definite iff $a > 0$ and $h^2 < ab$. (5)

Q.4 (a) Diagonalize the following quadratic form using Lagrange's Method

$$2x^2 + 2y^2 + 3z^2 - 4yz + 2xy - 4zx\ .$$ (8)

(b)Prove that the rank of a quadratic form is invariant under a non-singular linear transformation. (7)

PART -C

Q.5 (a) State and prove Cauchy Schwarz Inequality. (9)

(b) Verify Cauchy Schwarz Inequality for $u = (1,2,-2) and\ v(2,3,6) \in\ R^3$. (6)

Q.6 (a) Using Gram-Schmidt orthogonalization Process, Find an orthonormal basis of R^3(R)with standard inner product given the basis $\{(1,0,1),(1,0,-1),(0,3,4)\}$ (8)

(b) If W1 and W2 are subspaces of a finite dimensional inner product space, then Prove

$(w_1 + w_2)^{\perp} = w_1^{\perp} \cap w_2^{\perp}$ (7)

Q.7(a) State and prove Bessel's inequality for a finite dimensional inner product space. (9)

(b)Explain any two properties of an Adjoint operator of a Linear Transformation. (6)

****