MANAV RACHNA vidyanatariksha

MANAV RACHNA UNIVERSITY

(FORMERLY MANAV RACHNA COLLEGE OF ENGINEERING
NAAC ACCREDITED 'A' GRADE INSTITUTION)

Declared as State Private University under section 2f of the UGC act, 1956

DEPARTMENT OF MATHEMATICS

"T3, Examination, MAY-2018"

Semester: II
Subject: Complex Analysis
Branch: MATHEMATICS
Course Type: Core
Time: 3 Hours
Max.Marks: 100

Date of Exam: 24/05/2018
Subject Code: MAH511-T
Session: I
Course Nature: Hard
Program: M.Sc
Signatures: **HOD/Associate HOD:**

NOTE: Attempt any two questions from each part.

PART-A

1(a). Write down the conditions of analytic function. Also find f (z) if u(x, y) = cos x cosh y

(b). Evaluate $\int_0^{1+i} (x^2 - iy)\, dz$ along the paths (a). y = x (b). $y = x^2$

2 (a). Using the residue theorem, evaluate $\int_c \frac{e^x dz}{z(2+z)^2}$, where c is the circle $|z| = 2.5$

(b). Prove that $\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta} = \frac{2\pi}{\sqrt{1-a^2}}$, $a^2 < 1$

3 (a). State and prove Cauchy's Residue theorem.

(b0. State and prove C R equation in polar coordinates.

PART-B

4 (a). Define linear fractional transformation. Prove that "Every bilinear transformation is the resultant of bilinear transformation with simple geometric imports".

(b).Define cross ration. Also prove that the cross ratio remains invariant under a bilinear transformation.

5 (a). Prove that the mapping w = f (z) is conformal, then show that f(z) is an analytic function of z.

(b). Show that the transformation $w = \frac{2z+3}{z-4}$ transforms the circle $x^2 + y^2 - 4x = 0$ in to the straight line 4 u +3 =0 and explain the curve obtained is not a circle.

6 (a). Find the function that maps the upper help of the z-plane on to the interior of a triangle in the w-plane.

(b). Explain the Schwarz – Christoffel transformation.

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PART-C

7 (a). Explain the following with examples

(i). Analytic Continuation (ii). Complete analytic function

(b). State and prove Schwart's Reference Principle.

8 (a). State and prove Uniqueness of analytic continuation theorem.

(b). Show that the circle of convergence of the power series $\sum_{n=0}^{\infty} z^n$ is a natural boundary for its sum function.

9 (a).Show that the function $f_1(z) = \int_0^{\infty} e^{-zt} dt$ can be continued analytically. Also construct a power series which is analytic continuation of $f_1(z)$.

(b). If $f(\bar{z})$ = f(z) then prove that f(x) is real.

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